

# Which Bosonic Loop Corrections are Tested in Electroweak Precision Measurements?<sup>†‡</sup>

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## Abstract

The nature of the electroweak bosonic loop corrections to which current precision experiments are sensitive is explored. The set of effective parameters  $\Delta x$ ,  $\Delta y$ , and  $\varepsilon$ , which quantify SU(2) violation in an effective Lagrangian, is shown to be particularly useful for this purpose. The standard bosonic corrections are sizable only in the parameter  $\Delta y$ , while  $\Delta x$  and  $\varepsilon$  are sufficiently well approximated by the pure fermion-loop prediction. By analyzing the contributions to  $\Delta y$  it is shown that the bosonic loop corrections resolved by the present precision data are induced by the change in energy scale between the low-energy process muon decay and the energy scale of the LEP1 observables. If the (theoretical value of the) leptonic width of the W boson is used as input parameter instead of the Fermi constant  $G_\mu$ , no further bosonic loop corrections are necessary for compatibility between theory and experiment.

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# 1 Introduction

By comparing the results of precision experiments with the theoretical predictions of the electroweak Standard Model (SM) in various approximations, it is possible to test the loop corrections of this model, i.e. to test the SM as a quantized field theory. Considering the present experimental accuracy, the question then naturally arises to which radiative corrections the data are in fact sensitive. While the pure fermion-loop predictions of the SM involve only couplings that have already been studied in low-energy experiments (except for the couplings of the top quark), the full one-loop predictions of the SM involve also the non-abelian gauge structure and the Higgs sector, for which much less direct experimental information is available.

Genuine precision tests of the electroweak theory therefore require an experimental accuracy that allows to distinguish between the pure fermion-loop and the full one-loop predictions of the theory [1]. This accuracy was first reached in 1994, as shown in an analysis [2, 3] incorporating as observables the W-boson mass  $M_{W^\pm}$  and the leptonic Z-peak observables  $\Gamma_1$ , i.e. the leptonic Z-boson decay width, and  $\bar{s}_W^2$ , i.e. the leptonic effective weak mixing angle, which are not influenced by the discrepancies noted in certain hadronic decay modes of the Z boson. Indeed, by systematically discriminating between fermion-loop (vacuum-polarization) corrections to the  $\gamma$ , Z and  $W^\pm$  propagators and the full one-loop results, it was found that contributions beyond fermion loops are required for consistency with the experimental results on these observables. While the pure fermion-loop predictions were shown to be incompatible with the data, the complete one-loop prediction of the SM provides a consistent description of the experimental results. This implies that the data have become sensitive to bosonic radiative corrections and thus provide quantitative tests of the non-abelian gauge structure of the standard electroweak theory. The experimental evidence for bosonic loop corrections was also explored for the single observable  $\bar{s}_W^2$  in Ref. [4] and for  $M_{W^\pm}$  in Ref. [5]. The evidence for radiative corrections beyond the  $\alpha(M_Z^2)$ -Born approximation, which takes into account fermion-loop corrections to the photon propagator only, was explored in Ref. [6].

The analysis in Refs. [2, 3] is based on an effective Lagrangian [7] for electroweak interactions that incorporates possible sources of SU(2) violation in the leptonic sector via three effective parameters,  $\Delta x$ ,  $\Delta y$ , and  $\varepsilon$ . They parametrize SU(2) breaking in the vector-boson masses, in the couplings of the vector bosons to charged leptons and in the mixing among the neutral vector bosons. In the analysis based on this effective Lagrangian the parameters  $\Delta x$ ,  $\Delta y$ , and  $\varepsilon$  are directly related to observables and are thus manifestly gauge-independent quantities. Their theoretical predictions incorporate the full SM radiative corrections. This set of parameters is related by linear combinations to the parameters  $\varepsilon_i$  ( $i = 1, 2, 3$ ) of Ref. [8], which were introduced by isolating the leading terms of the top-quark mass dependence. Apart from emerging naturally from symmetry breaking in an underlying effective Lagrangian, the parameters  $\Delta x$ ,  $\Delta y$ , and  $\varepsilon$  are particularly convenient for investigating the relevance of radiative corrections beyond fermion loops. The evaluation of the parameters in the SM shows that the bosonic loop corrections required for consistency with the data are completely contained in only one of the parameters, namely  $\Delta y$ , while  $\Delta x$  and  $\varepsilon$  may consistently be approximated by the pure fermion-loop predictions. While  $\Delta y$  is at present the only parameter in which standard bosonic contributions

are significant, it is totally insensitive to the Higgs sector of the SM; it does not even show a logarithmic dependence for a heavy Higgs-boson mass.

In Ref. [9] the bosonic contributions to  $\Delta y$  have further been investigated. It has been shown that the bosonic corrections to which current precision experiments are sensitive can be traced back to a scale-change effect related to the use of the low-energy parameter  $G_\mu$ , which is measured in muon decay, for the analysis of the LEP observables. This fact has explicitly been demonstrated by inserting the SM theoretical value of the leptonic W-boson width as input instead of  $G_\mu$ .

In the present article the aforementioned results are surveyed. The investigations are based on the most recent data presented at the 1995 Summer Conferences [10]. The paper is organized as follows: In Sect. 2 the concept of analyzing the data in terms of the effective parameters  $\Delta x$ ,  $\Delta y$ , and  $\varepsilon$  is briefly sketched. The fermion loop predictions for these parameters are compared to the full SM predictions and to the experimental values of these parameters. In Sect. 3 it is shown that the bosonic contribution to  $\Delta y$  is strongly dominated by the correction induced by the change in energy scale from muon decay to the LEP observables. Supplementing the pure fermion-loop predictions with the bosonic scale-change contributions to  $\Delta y$  leads to a consistent description of the observables  $\Gamma_1$ ,  $\bar{s}_W^2$ , and  $M_{W^\pm}$ . In Sect. 4 the significance of the bosonic corrections is discussed in a scheme where the leptonic W-boson width is taken from theory and used as an input parameter instead of  $G_\mu$ . Final conclusions are drawn in Sect. 5.

## 2 Data analysis in terms of the effective parameters $\Delta x$ , $\Delta y$ , and $\varepsilon$

The effective Lagrangian introduced for the analysis of LEP1 observables in Refs. [2, 3, 7] quantifies SU(2)-breaking effects in the leptonic sector by the parameters  $x = (1 + \Delta x)$ ,  $y = (1 + \Delta y)$ , and  $\varepsilon$ . It contains the SM tree-level form of the vector-boson fermion interactions in the limit  $\Delta x = \Delta y = \varepsilon = 0$ .

In the charged-current Lagrangian the  $W^\pm$  boson is coupled to the weak isospin current  $j_\mu^\pm$  via the coupling  $g_{W^\pm}$ ,

$$\mathcal{L}_C = -\frac{1}{2}W^{+\mu\nu}W_{\mu\nu}^- - \frac{g_{W^\pm}}{\sqrt{2}}(j_\mu^+W^{+\mu} + h.c.) + M_{W^\pm}^2W_\mu^+W^{-\mu}. \quad (1)$$

In the transition to the neutral-current sector SU(2) symmetry is broken in the coupling of the  $(W^\pm, W^0)$  triplet by introducing the parameter  $y$ ,

$$g_{W^\pm}^2 = yg_{W^0}^2 = (1 + \Delta y)g_{W^0}^2, \quad (2)$$

and in the mass terms via the parameter  $x$ ,

$$M_{W^\pm}^2 = xM_{W^0}^2 = (1 + \Delta x)M_{W^0}^2. \quad (3)$$

SU(2) violation in  $\gamma W^0$  mixing is furthermore quantified by the parameter  $\varepsilon$  according to

$$\mathcal{L}_{\text{mix}} = -\frac{1}{2}\frac{e(M_Z^2)}{g_{W^0}}(1 - \varepsilon)A_{\mu\nu}W^{0,\mu\nu}, \quad (4)$$

where  $e^2(M_Z^2) \equiv 4\pi\alpha(M_Z^2)$  denotes the electromagnetic coupling at the Z-boson mass. In Refs. [2, 3, 7] the charged-current coupling  $g_{W^\pm}$  was defined with respect to muon decay, i.e. at a low-energy scale, as

$$g_{W^\pm}^2 \equiv g_{W^\pm}^2(0) \equiv 4\sqrt{2}G_\mu M_{W^\pm}^2, \quad (5)$$

while in the neutral sector  $g_{W^0}^2 \equiv g_{W^0}^2(M_Z^2)$  corresponds to the coupling at the Z-boson scale.

After diagonalization the neutral-current Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_N = & -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{M_{W^\pm}^2}{2x'(1-\bar{s}_W^2(1-\varepsilon'))}Z_\mu Z^\mu \\ & - \frac{g_{W^\pm}}{\sqrt{y'(1-\bar{s}_W^2(1-\varepsilon'))}} \left[ j_\mu^3 - \bar{s}_W^2 j_{\text{em},\mu} \right] Z^\mu, \end{aligned} \quad (6)$$

where the shorthands

$$x' = x + 2s_0^2\delta, \quad y' = y - 2s_0^2\delta, \quad \varepsilon' = \varepsilon - \delta, \quad (7)$$

have been used and the small quantity  $\delta$  ( $\delta \sim 10^{-4}$  in the SM) describes parity violation in the photonic coupling at the Z-boson mass scale (see Ref. [3]). The effective weak mixing angle  $\bar{s}_W^2$ , which empirically is determined by the charged lepton asymmetry at the Z-boson resonance, is given as

$$\bar{s}_W^2 = \frac{e^2(M_Z^2)}{g_{W^\pm}^2(0)} y'(1-\varepsilon'), \quad (8)$$

and  $s_0^2$  in (7) is defined via

$$s_0^2(1-s_0^2) = s_0^2 c_0^2 = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_\mu M_Z^2}. \quad (9)$$

Using the effective Lagrangian  $\mathcal{L} = \mathcal{L}_C + \mathcal{L}_N$  to express the weak mixing angle  $\bar{s}_W^2$ , the  $W^\pm$  mass  $M_{W^\pm}$ , and the leptonic Z-boson width  $\Gamma_1$  in terms of  $\Delta x$ ,  $\Delta y$ , and  $\varepsilon$ , and linearizing in these parameters yields

$$\begin{aligned} \bar{s}_W^2 &= s_0^2 \left[ 1 - \frac{1}{c_0^2 - s_0^2} \varepsilon - \frac{c_0^2}{c_0^2 - s_0^2} (\Delta x - \Delta y) + (c_0^2 - s_0^2) \delta \right], \\ \frac{M_{W^\pm}}{M_Z} &= c_0 \left[ 1 + \frac{s_0^2}{c_0^2 - s_0^2} \varepsilon + \frac{c_0^2}{2(c_0^2 - s_0^2)} \Delta x - \frac{s_0^2}{2(c_0^2 - s_0^2)} \Delta y \right], \\ \Gamma_1 &= \Gamma_1^{(0)} \left[ 1 + \frac{8s_0^2}{1 + (1 - 4s_0^2)^2} \left\{ \frac{1 - 4s_0^2}{c_0^2 - s_0^2} \varepsilon + \frac{c_0^2 - s_0^2 - 4s_0^4}{4s_0^2(c_0^2 - s_0^2)} (\Delta x - \Delta y) + 2s_0^2 \delta \right\} \right], \end{aligned} \quad (10)$$

with

$$\Gamma_1^{(0)} = \frac{\alpha(M_Z^2)M_Z}{48s_0^2c_0^2} \left[ 1 + (1 - 4s_0^2)^2 \right] \left( 1 + \frac{3\alpha}{4\pi} \right). \quad (11)$$

As experimental input for our analysis we use the most recent experimental data [10],

$$\begin{aligned}
M_Z &= 91.1884 \pm 0.0022 \text{ GeV}, \\
\Gamma_1 &= 83.93 \pm 0.14 \text{ MeV}, \\
\bar{s}_w^2(\text{LEP}) &= 0.23186 \pm 0.00034, \\
\frac{M_{W^\pm}}{M_Z}(\text{UA2} + \text{CDF}) &= 0.8802 \pm 0.0018, \\
\alpha_s(M_Z^2) &= 0.123 \pm 0.006.
\end{aligned} \tag{12}$$

We restrict the analysis to the LEP value of  $\bar{s}_w^2$ . Using instead the combined LEP+SLD value,  $\bar{s}_w^2 = 0.23143 \pm 0.00028$  [10], does not significantly affect our results. The data in (12) are supplemented by the Fermi constant

$$G_\mu = 1.16639(2) \cdot 10^{-5} \text{ GeV}^{-2}, \tag{13}$$

and the electromagnetic coupling at the Z-boson resonance,

$$\alpha(M_Z^2)^{-1} = 128.89 \pm 0.09, \tag{14}$$

which was taken from the recent updates [11] of the evaluation of the hadronic vacuum polarization.

Using these input data and solving (11) for  $\Delta x$ ,  $\Delta y$ , and  $\varepsilon$  with a corresponding error analysis yields as experimental values for the parameters

$$\begin{aligned}
\Delta x^{\text{exp}} &= (10.1 \pm 4.2 \pm 0.2) \times 10^{-3}, \\
\Delta y^{\text{exp}} &= (5.4 \pm 4.3 \pm 0.2) \times 10^{-3}, \\
\varepsilon^{\text{exp}} &= (-5.3 \pm 1.6 \mp 0.5) \times 10^{-3}.
\end{aligned} \tag{15}$$

The first errors indicated in (15) are due to the statistical and systematic errors in the experimental data, and the second errors give the deviations caused by the replacement  $\alpha(M_Z^2)^{-1} \rightarrow \alpha(M_Z^2)^{-1} \pm \delta\alpha(M_Z^2)^{-1}$  according to (14).

In Fig. 1 the experimental results for the parameters  $\Delta x$ ,  $\Delta y$ , and  $\varepsilon$  are compared with the theoretical predictions of the full SM and with the pure fermion-loop predictions. In the theoretical predictions the leading two-loop contributions of order  $\mathcal{O}(\alpha_s \alpha t)$  and  $\mathcal{O}(\alpha^2 t^2)$  have also been included (see Ref. [3]). The arrow in the upper left corner of each figure indicates how the empirical 83% C.L. (i.e.  $1.9\sigma$ ) ellipse in the corresponding plane of  $(\Delta x, \Delta y, \varepsilon)$ -space is shifted by the replacement  $\alpha(M_Z^2)^{-1} \rightarrow \alpha(M_Z^2)^{-1} + \delta\alpha(M_Z^2)^{-1}$ . In the theoretical predictions for the effective parameters the error in  $\alpha(M_Z^2)$  enters only in higher orders and is therefore negligible.

Figure 1 shows that the experimental results are in excellent agreement with the full SM predictions for top-quark masses which are compatible with the value of  $m_t = 180 \pm 12 \text{ GeV}$  [12] resulting from the experimental detection at CDF and DØ. For the parameters  $\Delta x$  and  $\varepsilon$  the data are consistently described by the pure fermion-loop predictions alone, i.e. omission of the SM bosonic corrections does not lead to a deviation from the experimental results for these parameters. For the parameter  $\Delta y$ , on the other hand, the pure fermion-loop prediction alone does not yield a consistent description of the data, i.e. in this

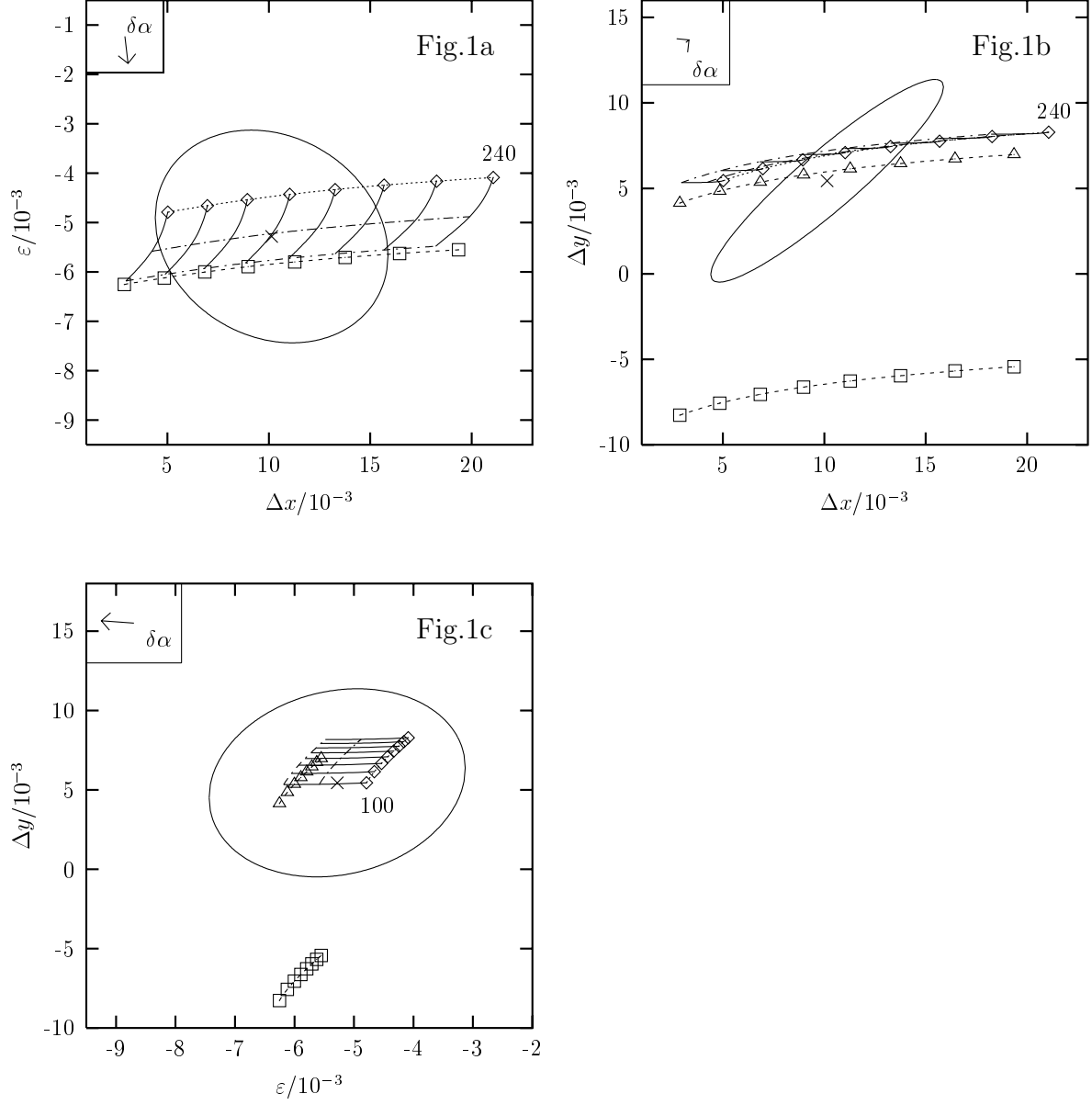


Figure 1: Comparison of the 83% C.L. ( $1.9\sigma$ ) ellipses of the experimental data with the full SM predictions and the pure fermion-loop predictions in each plane of the three-dimensional  $(\Delta x, \Delta y, \varepsilon)$ -space. The full SM predictions are shown for Higgs-boson masses of 100 GeV (dotted with diamonds), 300 GeV (long-dashed-dotted), 1 TeV (short-dashed-dotted) parametrized by the top-quark mass ranging from 100-240 GeV in steps of 20 GeV. The pure fermion-loop predictions (short-dashed curves with squares) are shown for the same top-quark masses. The additional dashed curves with triangles correspond to  $(\Delta y_{\text{ferm}} + \Delta y_{\text{bos}}^{\text{SC}})$  (see Sect. 3). The arrow in the upper left corner of each figure indicates how the empirical ellipse is shifted by the replacement  $\alpha(M_Z^2)^{-1} \rightarrow \alpha(M_Z^2)^{-1} + \delta\alpha(M_Z^2)^{-1}$ .

parameter additional corrections, such as the standard bosonic ones, are required for an agreement between theory and experiment. Since  $\Delta y_{\text{ferm}}$  can reliably be calculated from the experimentally well-known couplings of leptons and quarks, this shows that the data have indeed become sensitive to bosonic radiative corrections. These significant bosonic corrections are localized in the single effective parameter  $\Delta y$ . It is interesting to note that  $\Delta y$  is totally insensitive to the Higgs sector of the SM. It does not even show a logarithmic dependence for a large Higgs-boson mass (see Refs. [9, 13]).

The extra curves with triangles included in Figs. 1b and 1c indicate the sum ( $\Delta y_{\text{ferm}} + \Delta y_{\text{bos}}^{\text{SC}}$ ), which besides the fermion-loop contributions also includes the bosonic corrections associated with the change in energy scale from muon decay to W-boson decay, as will be discussed in Sect. 3.

### 3 Scale-change and isospin-breaking contributions to the parameter $\Delta y$

As explained in the last section (see (2) and (5)), the effective parameter  $\Delta y$ , which incorporates the sizable bosonic loop corrections, describes both the change from 0 to  $M_Z$  in the energy scale and the transition from the charged-current to the neutral-current coupling. These two effects can separately be investigated by introducing the charged-current coupling  $g_{W^\pm}(M_{W^\pm}^2)$  at the W-boson mass shell. This coupling may be defined via the leptonic width  $\Gamma_1^W$  of the W boson,

$$g_{W^\pm}^2(M_{W^\pm}^2) \equiv \frac{48\pi}{M_{W^\pm}} \Gamma_1^W \left(1 + c_0^2 \frac{3\alpha}{4\pi}\right)^{-1}. \quad (16)$$

In analogy to (2) we relate  $g_{W^\pm}(M_{W^\pm}^2)$  to  $g_{W^0}(M_Z^2)$  by a parameter  $\Delta y^{\text{IB}}$ ,

$$g_{W^\pm}^2(M_{W^\pm}^2) = y^{\text{IB}} g_{W^0}^2(M_Z^2) = (1 + \Delta y^{\text{IB}}) g_{W^0}^2(M_Z^2), \quad (17)$$

where the index “IB” refers to weak “isospin-breaking”. In (16) we have introduced a factor  $(1 + c_0^2 3\alpha/(4\pi))$  by convention. It is related to the convention chosen in the treatment of the photonic corrections to the leptonic Z-boson decay width  $\Gamma_1$  (see (11)). The photonic contributions to  $\Gamma_1$  are pure QED corrections giving rise to a factor  $(1 + 3\alpha/(4\pi))$  that is split off and not included in  $\Delta x$ ,  $\Delta y$ , and  $\varepsilon$ . In order to treat the photonic corrections on the same footing in both the neutral and charged vector boson decay, one has to split off an analogous correction factor also for the decay of the W boson (see Ref. [9]). The appearance of  $c_0^2$  in the correction factor of (16) is due to the rotation in isospin space relating the physical field  $Z$  to the field  $W^0$  entering the SU(2) isotriplet. Numerically the correction term introduced in (16) amounts to  $c_0^2 3\alpha/(4\pi) = 1.3 \times 10^{-3}$ . Even though the convention chosen in (16) is well justified, it is worth noting that a different treatment of the photonic corrections, e.g. omission of the correction factor in (16), would only lead to minor changes that do not influence our final conclusions.

The transition from the charged-current coupling at the scale of the muon mass,  $g_{W^\pm}(0)$ , to the charged-current coupling obtained from the decay of the W boson into leptons,  $g_{W^\pm}(M_{W^\pm}^2)$ , can be expressed by a parameter  $\Delta y^{\text{SC}}$ ,

$$g_{W^\pm}^2(0) = y^{\text{SC}} g_{W^\pm}^2(M_{W^\pm}^2) = (1 + \Delta y^{\text{SC}}) g_{W^\pm}^2(M_{W^\pm}^2), \quad (18)$$

$m_t/\text{GeV}$	$\Delta y_{\text{ferm}}/10^{-3}$	$\Delta y_{\text{ferm}}^{\text{SC}}/10^{-3}$	$\Delta y_{\text{ferm}}^{\text{IB}}/10^{-3}$
120	-7.57	-7.42	-0.15
180	-6.27	-7.79	1.52
240	-5.44	-7.90	2.46

$M_H/\text{GeV}$	$\Delta y_{\text{bos}}/10^{-3}$	$\Delta y_{\text{bos}}^{\text{SC}}/10^{-3}$	$\Delta y_{\text{bos}}^{\text{IB}}/10^{-3}$
100	13.72	12.47	1.25
300	13.62	12.42	1.20
1000	13.61	12.41	1.20

Table 1: Fermionic and bosonic contributions to  $\Delta y$ ,  $\Delta y^{\text{SC}}$ , and  $\Delta y^{\text{IB}}$  for different values of  $m_t$  and  $M_H$ .

where the index “SC” means “scale change”. Inserting (17) into (18) and comparing with (2), one finds that in linear approximation the parameter  $\Delta y$  is split into two additive contributions,

$$\Delta y = \Delta y^{\text{SC}} + \Delta y^{\text{IB}}, \quad (19)$$

which furnish the transition from  $g_{W^\pm}^2(0)$  to  $g_{W^\pm}^2(M_{W^\pm}^2)$  and from  $g_{W^\pm}^2(M_{W^\pm}^2)$  to  $g_{W^0}^2(M_Z^2)$ , respectively. Upon substituting (5) and (16) in (18), one finds

$$\Delta y^{\text{SC}} = \frac{M_{W^\pm}^3 G_\mu}{6\sqrt{2}\pi\Gamma_1^W} - 1 + c_0^2 \frac{3\alpha}{4\pi}, \quad (20)$$

which allows to determine  $\Delta y^{\text{SC}}$  (and consequently also  $\Delta y^{\text{IB}}$ ) both experimentally and theoretically.

It should be noted at this point that the scale-change effect discussed here does not correspond to an ordinary “running” of universal (propagator-type) contributions, as  $g_{W^\pm}(0)$  and  $g_{W^\pm}(M_{W^\pm}^2)$ , being defined with reference to muon decay and W-boson decay, respectively, are obviously process-dependent quantities. Accordingly, the bosonic contributions to  $\Delta y^{\text{SC}}$  (and also to  $\Delta y$  and  $\Delta y^{\text{IB}}$ ) are process-dependent. As these three parameters are directly related to observables, i.e. to complete S-matrix elements, they are manifestly gauge-independent.

The analytical results of the SM predictions for  $\Delta y^{\text{SC}}$  and  $\Delta y^{\text{IB}}$  have been given in Ref. [9]. As it is the case for  $\Delta y_{\text{bos}}$ , also  $\Delta y_{\text{bos}}^{\text{SC}}$  and  $\Delta y_{\text{bos}}^{\text{IB}}$  are insensitive to variations in the Higgs-boson mass  $M_H$ . In Tab. 1 numerical results for the fermionic and bosonic contributions to  $\Delta y$ ,  $\Delta y^{\text{SC}}$ , and  $\Delta y^{\text{IB}}$  are given for different values of  $m_t$  and  $M_H$ . The table shows that both  $\Delta y_{\text{ferm}}$  and  $\Delta y_{\text{bos}}$  are dominated by the scale-change contributions  $\Delta y_{\text{ferm}}^{\text{SC}}$  and  $\Delta y_{\text{bos}}^{\text{SC}}$ , respectively. As these contributions enter with different signs, there are strong cancellations in  $\Delta y^{\text{SC}}$  and  $\Delta y$ .

In Fig. 2 we have plotted the SM one-loop result for  $\Delta y$ ,  $\Delta y_{\text{ferm}}$ ,  $\Delta y_{\text{ferm}}^{\text{SC}}$ , and  $(\Delta y_{\text{ferm}} + \Delta y_{\text{bos}}^{\text{SC}})$  as a function of  $m_t$  (using  $M_H = 300\text{ GeV}$ ). The error band in Fig. 2 indicates the experimental value of  $\Delta y$  given in (15). Fig. 2 first of all displays the above-mentioned fact that the pure fermion-loop contribution  $\Delta y_{\text{ferm}}$  is not sufficient to achieve agreement



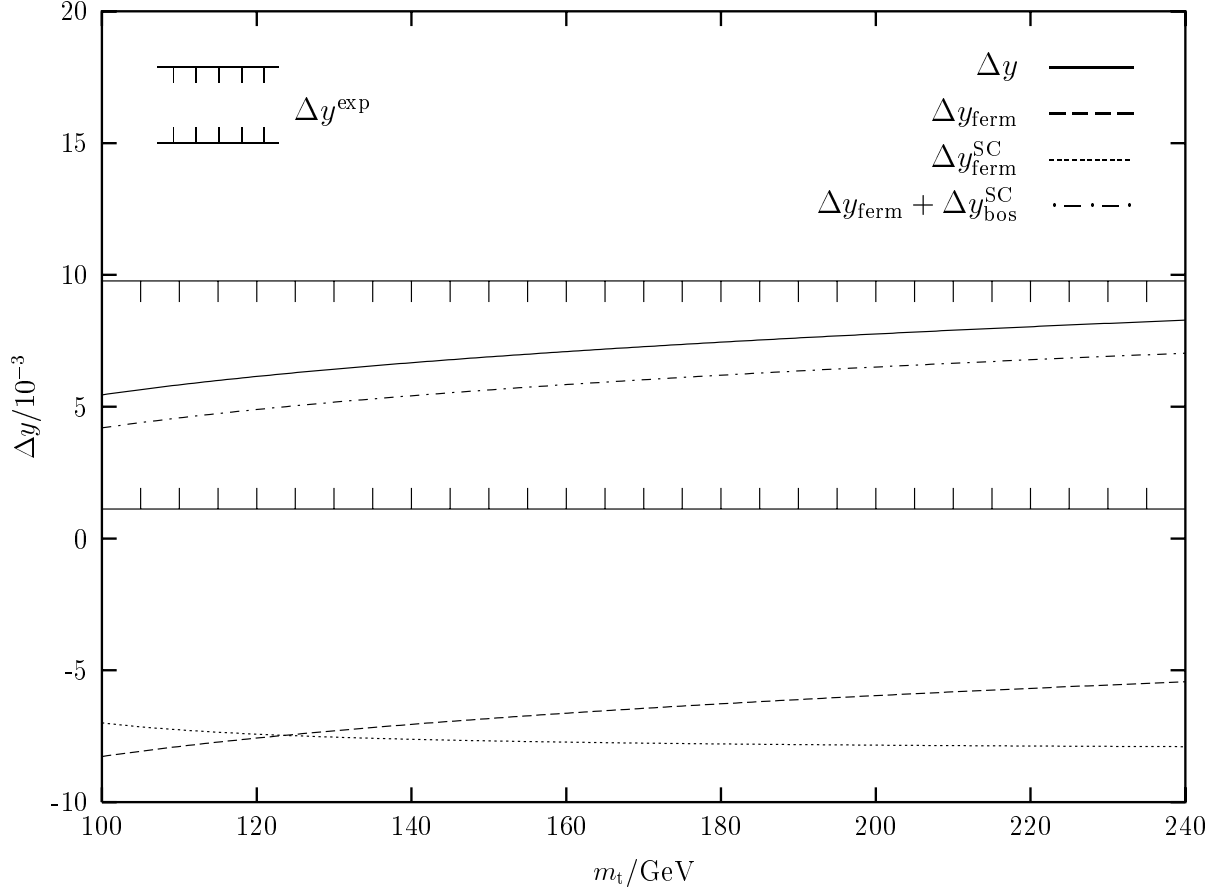


Figure 2: The one-loop SM predictions for  $\Delta y$ ,  $\Delta y_{\text{ferm}}$ ,  $\Delta y_{\text{ferm}}^{\text{SC}}$ , and  $(\Delta y_{\text{ferm}} + \Delta y_{\text{bos}}^{\text{SC}})$  as a function of  $m_t$ . The difference between the curves for  $\Delta y$  and  $(\Delta y_{\text{ferm}} + \Delta y_{\text{bos}}^{\text{SC}})$  corresponds to the small contribution of  $\Delta y_{\text{bos}}^{\text{IB}}$ . The experimental value of  $\Delta y$ ,  $\Delta y^{\text{exp}} = (5.4 \pm 4.3) \times 10^{-3}$ , is indicated by the error band.

with  $\Delta y^{\text{exp}}$ . The contribution of  $\Delta y_{\text{ferm}}^{\text{SC}}$  is approximately constant while  $\Delta y_{\text{ferm}} = \Delta y_{\text{ferm}}^{\text{SC}} + \Delta y_{\text{ferm}}^{\text{IB}}$  shows a  $\log(m_t)$ -dependence which enters through  $\Delta y_{\text{ferm}}^{\text{IB}}$  (see Ref. [9]). In contrast to  $\Delta y_{\text{ferm}}$ , the complete one-loop result,  $\Delta y = \Delta y_{\text{ferm}} + \Delta y_{\text{bos}}$ , is in perfect accordance with the data. Figure 2 furthermore visualizes that  $\Delta y_{\text{bos}}^{\text{SC}}$  constitutes by far the dominant part of the bosonic contributions to  $\Delta y$ . Combining the complete fermionic contribution  $\Delta y_{\text{ferm}}$  with the bosonic scale-change contribution  $\Delta y_{\text{bos}}^{\text{SC}}$  leads to a consistent description of the current precision data, while the contribution of  $\Delta y_{\text{bos}}^{\text{IB}}$  does not give rise to a significant effect.

As a consequence of these results on  $\Delta y$  and of the results of Sect. 2 on  $\Delta x$  and  $\varepsilon$  we find that the effective parameters  $\Delta x$ ,  $\Delta y$ ,  $\varepsilon$  are well approximated by

$$\Delta x \approx \Delta x_{\text{ferm}}, \quad \Delta y \approx \Delta y_{\text{ferm}} + \Delta y_{\text{bos}}^{\text{SC}}, \quad \varepsilon \approx \varepsilon_{\text{ferm}}, \quad (21)$$

i.e. supplementing the fermion-loop approximation by the bosonic scale-change contribution  $\Delta y_{\text{bos}}^{\text{SC}}$  to  $\Delta y$  leads to results that deviate from the complete one-loop prediction for the effective parameters by less than the experimental errors.

It is worth demonstrating this fact not only for the effective parameters but also explicitly for the observables  $\bar{s}_{\text{W}}^2$ ,  $\Gamma_1$ ,  $M_{\text{W}^\pm}/M_Z$ . This is done in Fig. 3, where the 68% C.L. ( $1.9\sigma$ ) volume defined by the most recent 1995 data (12) in the three-dimensional  $(M_{\text{W}^\pm}/M_Z, \bar{s}_{\text{W}}^2, \Gamma_1)$ -space is shown together with the full SM prediction, the pure fermion-loop prediction and the fermion-loop prediction supplemented by the bosonic contribution  $\Delta y_{\text{bos}}^{\text{SC}}$ . For completeness the  $\alpha(M_Z^2)$ -Born approximation is also indicated in the plots. The error bars shown for the  $\alpha(M_Z^2)$ -Born approximation also apply to all other theoretical predictions. They originate from the errors of the input parameters and are dominated by the error of  $\alpha(M_Z^2)$  given in (14). The projections of the 68% C.L. volume onto the planes of the three-dimensional  $(M_{\text{W}^\pm}/M_Z, \bar{s}_{\text{W}}^2, \Gamma_1)$ -space correspond to the 83% C.L. ellipse in each plane, while the projections onto the individual axes correspond to the 94% C.L. there.

In Fig. 3a the single line shows the theoretical prediction for values of  $m_t$  varying from  $m_t = 100 - 240$  GeV taking into account only fermion-loop corrections. The three connected lines show the full SM predictions. They correspond to  $M_{\text{H}} = 100, 300, 1000$  GeV, respectively, and  $m_t$  is again varied from  $m_t = 100 - 240$  GeV. In both theoretical predictions the leading two-loop contributions of order  $\mathcal{O}(\alpha_s \alpha t)$  and  $\mathcal{O}(\alpha^2 t^2)$  have been included (see Ref. [3]). The full SM prediction is in agreement with the data for the empirical value of the top-quark mass,  $m_t = 180 \pm 12$  GeV [12]. The pure fermion-loop prediction, on the other hand, differs from the data by several standard deviations, which clearly illustrates the sensitivity of the data to SM bosonic loop effects. For  $m_t = 180$  GeV the pure fermion-loop predictions at one-loop order read

$$\begin{aligned}\bar{s}_{\text{W},\text{ferm}}^2 &= 0.22747 \mp 0.00023, \\ \left(\frac{M_{\text{W}^\pm}}{M_Z}\right)_{\text{ferm}} &= 0.88358 \pm 0.00013, \\ \Gamma_{1,\text{ferm}} &= 85.299 \pm 0.012 \text{ MeV},\end{aligned}\tag{22}$$

which deviate from the experimental values by  $-13\sigma$ ,  $1.9\sigma$  and  $9.8\sigma$ , respectively. As mentioned above, all uncertainties of theoretical predictions are dominated by the error of  $\alpha(M_Z^2)$  given in (14). The  $\alpha(M_Z^2)$ -Born approximations for the observables read

$$\begin{aligned}s_0^2 &= 0.23112 \mp 0.00023, \\ c_0 &= 0.87686 \pm 0.00013, \\ \Gamma_1^{(0)} &= 83.563 \pm 0.012 \text{ MeV},\end{aligned}\tag{23}$$

corresponding to a deviation of  $-2.2\sigma$ ,  $-1.9\sigma$  and  $-2.6\sigma$ , respectively, from the experimental data. The fact that the values in (23) are closer to the empirical data and the full SM predictions than the fermion-loop prediction (22) is a consequence of the cancellation between fermionic and bosonic contributions in the single parameter  $\Delta y$ .

Figure 3b shows the theoretical predictions obtained by combining the pure fermion-loop prediction with the bosonic contribution  $\Delta y_{\text{bos}}^{\text{SC}}$  according to (21). As expected from Fig. 2, adding only the bosonic scale-change contribution  $\Delta y_{\text{bos}}^{\text{SC}}$  to the fermion-loop contribution is sufficient to obtain a theoretical prediction that is in agreement with the data.

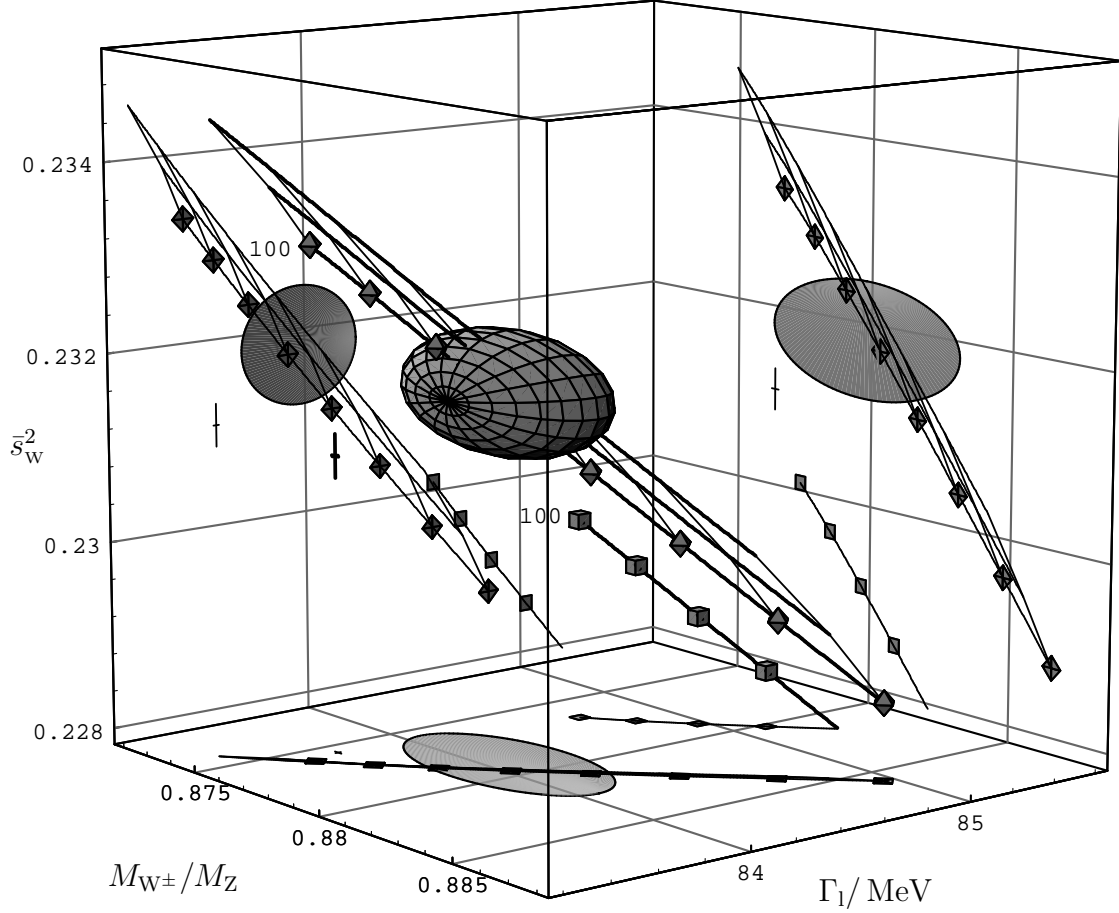


Figure 3a: Three-dimensional plot of the 68% C.L. ( $1.9\sigma$ ) ellipsoid of the experimental data in  $(M_{W^\pm}/M_Z, \bar{s}_W^2, \Gamma_1)$ -space and comparison with the full SM prediction (connected lines) and the pure fermion-loop prediction (single line with cubes). The full SM prediction is shown for Higgs-boson masses of  $M_H = 100$  GeV (line with diamonds), 300 GeV, and 1 TeV parametrized by  $m_t$  ranging from 100–240 GeV in steps of 20 GeV. In the pure fermion-loop prediction the cubes also indicate steps in  $m_t$  of 20 GeV starting with  $m_t = 100$  GeV. The cross outside the ellipsoid indicates the  $\alpha(M_Z^2)$ -Born approximation with the corresponding error bars, which also apply to all other theoretical predictions.

It is very close to the full SM prediction, i.e. the difference between these predictions is below experimental resolution.

We therefore conclude that the bosonic corrections required for consistency between the precision data for the observables  $\bar{s}_W^2$ ,  $\Gamma_1$ ,  $M_{W^\pm}/M_Z$  and the theoretical predictions in terms of the input parameters  $\alpha(M_Z^2)$ ,  $M_Z$  and  $G_\mu$  are just those contained in the parameter  $\Delta y_{\text{bos}}^{\text{SC}}$ . As  $\Delta y^{\text{SC}}$  describes the transition between  $g_{W^\pm}^2(0)$ , i.e.  $G_\mu$ , and  $g_{W^\pm}^2(M_{W^\pm}^2)$ , i.e.  $\Gamma_1^W$ , the bosonic corrections required by the precision data can be identified as an effect of

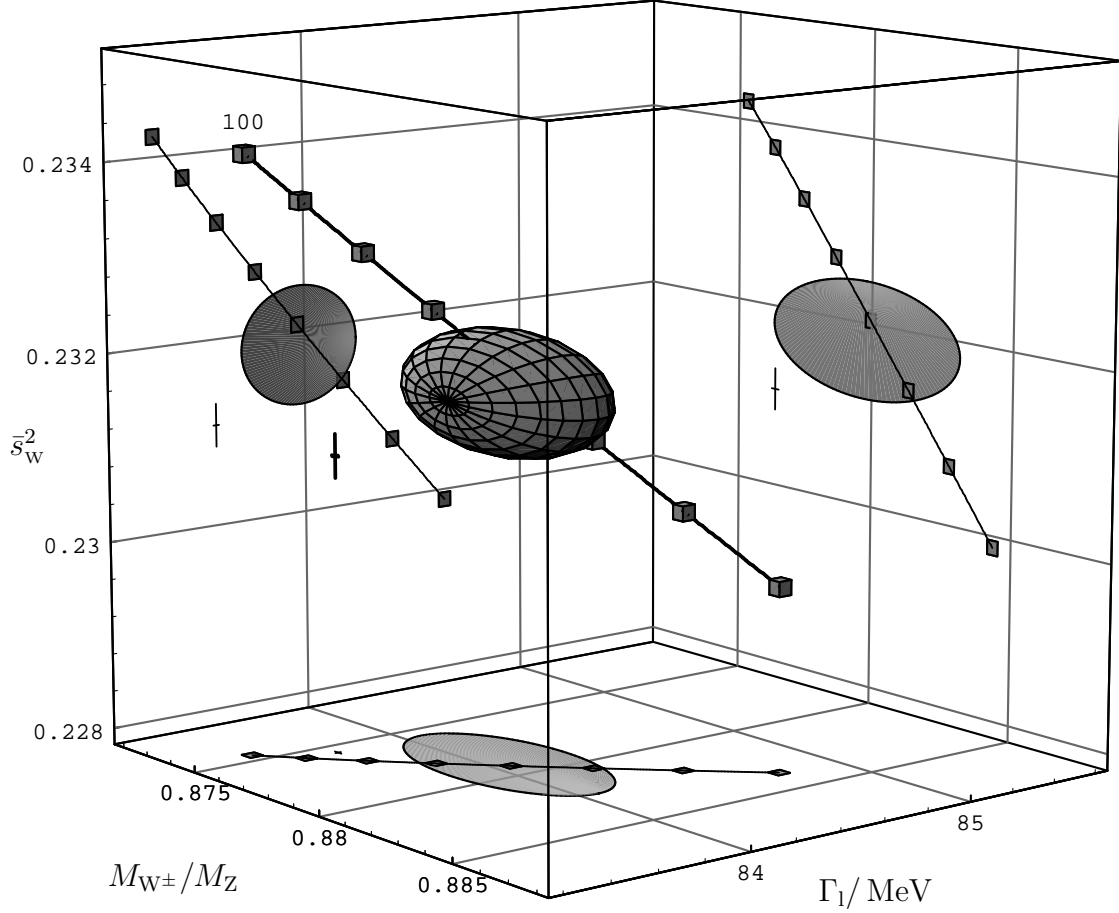


Figure 3b: Three-dimensional plot of the 68% C.L. ( $1.9\sigma$ ) ellipsoid of the experimental data in  $(M_{W^\pm}/M_Z, \bar{s}_W^2, \Gamma_1)$ -space and comparison with the theoretical prediction obtained by combining the fermion-loop contribution with the bosonic correction  $\Delta y_{\text{bos}}^{\text{SC}}$  related to the scale change from  $G_\mu$  to  $\Gamma_1^W$ . The theoretical prediction is parametrized by  $m_t$  ranging from 100–240 GeV in steps of 20 GeV.

changing the energy scale from the low-energy process muon decay to the energy scale of W-boson decay. All other bosonic effects, in particular the  $\log(M_H)$ -dependent vacuum-polarization contributions contained in  $\Delta x_{\text{bos}}$  and  $\varepsilon_{\text{bos}}$ , are below experimental resolution for Higgs-boson masses in the perturbative regime, i.e. below  $\sim 1$  TeV.

The sensitivity of the data on variations in the Higgs-boson mass can be inspected in Fig. 3a. If  $m_t$  is fixed, the intersection of the three-dimensional 68% C.L. ( $1.9\sigma$ ) volume with the lines representing the full SM prediction shows a certain sensitivity to the Higgs-boson mass. It can also be seen, however, that in the direction in three-dimensional space in which the  $M_H$ -dependence (for fixed  $m_t$ ) is sizable also the uncertainty in the theoretical predictions due to the error in  $\alpha(M_Z^2)$  is large.

## 4 Radiative corrections in the $\Gamma_1^W$ -scheme

After having identified the source of the important bosonic corrections in the analysis of the precision data as a scale-change effect related to the use of the low-energy input parameter  $G_\mu$ , it is evident that these large bosonic corrections could be avoided by expressing the theoretical predictions for the observables  $\bar{s}_W^2$ ,  $\Gamma_1$ ,  $M_{W^\pm}/M_Z$  in terms of input parameters being defined at the scale of the vector-boson masses. This can be achieved by using the W-boson width  $\Gamma_1^W$  instead of the Fermi constant  $G_\mu$  as an input parameter.

In the language of the effective Lagrangian  $\mathcal{L}_C$  given in (1) the use of the input quantity  $\Gamma_1^W$  instead of  $G_\mu$  means that the charged-current coupling in (1) is identified with  $g_{W^\pm}(M_{W^\pm}^2)$  defined via the W-boson width  $\Gamma_1^W$  (see (16)) rather than with  $g_{W^\pm}(0)$  defined via muon decay (see (5)). With this identification the transition between  $g_{W^\pm}(0)$  and  $g_{W^\pm}(M_{W^\pm}^2)$ , and accordingly the contribution of  $\Delta y^{\text{SC}}$ , does not occur. The radiative corrections to the observables  $\bar{s}_W^2$ ,  $\Gamma_1$ , and  $M_{W^\pm}/M_Z$  are completely contained in the parameters  $\Delta x$ ,  $\Delta y^{\text{IB}}$ , and  $\varepsilon$ , in which SM corrections beyond fermion loops do not give rise to significant contributions.

In this “ $\Gamma_1^W$ -scheme” the lowest-order values  $\hat{s}_0^2$ ,  $\hat{c}_0$ , and  $\hat{\Gamma}_1^{(0)}$  of the observables are given in terms of the input quantities  $\alpha(M_Z^2)$ ,  $M_Z$ , and  $\Gamma_1^W$  as

$$\frac{\hat{s}_0^2}{\hat{c}_0} \equiv \frac{\alpha(M_Z^2)M_Z}{12\Gamma_1^W} \left(1 + c_0^2 \frac{3\alpha}{4\pi}\right), \quad \hat{c}_0^2 \equiv (1 - \hat{s}_0^2), \quad (24)$$

and

$$\hat{\Gamma}_1^{(0)} = \frac{\alpha(M_Z^2)M_Z}{48\hat{s}_0^2\hat{c}_0^2} \left[1 + (1 - 4\hat{s}_0^2)^2\right] \left(1 + \frac{3\alpha}{4\pi}\right). \quad (25)$$

The linearized relations between the observables and the effective parameters  $\Delta x$ ,  $\Delta y^{\text{IB}}$ , and  $\varepsilon$  read

$$\begin{aligned} \bar{s}_W^2 &= \hat{s}_0^2 \left[1 + \frac{\hat{c}_0^2}{2 - \hat{s}_0^2} \Delta x + \frac{2\hat{c}_0^2}{2 - \hat{s}_0^2} \Delta y^{\text{IB}} + \frac{3\hat{s}_0^2 - 2}{2 - \hat{s}_0^2} \varepsilon + (\hat{c}_0^2 - \hat{s}_0^2) \delta\right], \\ \frac{M_{W^\pm}}{M_Z} &= \hat{c}_0 \left[1 + \frac{\hat{c}_0^2}{2 - \hat{s}_0^2} \Delta x - \frac{\hat{s}_0^2}{2 - \hat{s}_0^2} \Delta y^{\text{IB}} + \frac{2\hat{s}_0^2}{2 - \hat{s}_0^2} \varepsilon\right], \\ \Gamma_1 &= \hat{\Gamma}_1^{(0)} \left[1 - \frac{2}{(2 - \hat{s}_0^2)[1 + (1 - 4\hat{s}_0^2)^2]} \left((1 - 2\hat{s}_0^2 - 4\hat{s}_0^4)(\Delta x + 2\Delta y^{\text{IB}}) \right. \right. \\ &\quad \left. \left. - 2\hat{s}_0^2(1 - 10\hat{s}_0^2)\varepsilon - 8\hat{s}_0^4(2 - \hat{s}_0^2)\delta\right)\right]. \end{aligned} \quad (26)$$

The relations (26) could in principle be used for a data analysis of the observables  $\bar{s}_W^2$ ,  $\Gamma_1$ , and  $M_{W^\pm}/M_Z$  in the  $\Gamma_1^W$ -scheme, i.e. with  $\alpha(M_Z^2)$ ,  $M_Z$ , and  $\Gamma_1^W$  as experimental input quantities. Assuming (hypothetically) the same experimental accuracy as in the “ $G_\mu$ -scheme” (input parameters  $\alpha(M_Z^2)$ ,  $M_Z$ , and  $G_\mu$ ) and an experimental value of  $\Gamma_1^W$  being in agreement with the SM prediction, a consistent description of the data in the  $\Gamma_1^W$ -scheme would be possible by solely including the pure fermion-loop predictions in the effective parameters.

At present a data analysis using the  $\Gamma_1^W$ -scheme would of course not be sensible owing to the large experimental error in the determination of the W-boson width. From Ref. [14] we

	$G_\mu$ -scheme		$\Gamma_1^W$ -scheme		exp. error
	ferm. corr.	bos. corr.	ferm. corr.	bos. corr.	
$\frac{\Delta \bar{s}_W^2}{\bar{s}_W^2}/10^{-3}$	-15.8	16.3	11.0	1.3	1.5
$\frac{\Delta M_{W^\pm}}{M_{W^\pm}}/10^{-3}$	7.7	-1.6	3.7	0.6	2.0
$\frac{\Delta \Gamma_1}{\Gamma_1}/10^{-3}$	20.8	-14.3	-1.8	-1.7	1.7

Table 2: Relative size of the SM fermionic and bosonic one-loop corrections to the observables  $\bar{s}_W^2$ ,  $M_{W^\pm}$ , and  $\Gamma_1$  in the  $G_\mu$ -scheme (input parameters  $G_\mu$ ,  $M_Z$ , and  $\alpha(M_Z^2)$ ) and in the simulated  $\Gamma_1^W$ -scheme (input parameters  $\Gamma_1^W = 226.3$  MeV,  $M_Z$ , and  $\alpha(M_Z^2)$ ) for  $m_t = 180$  GeV and  $M_H = 300$  GeV. The relative experimental error of the observables is also indicated.

have  $\Gamma_T^{W,\text{exp}} = (2.08 \pm 0.07)$  GeV for the total decay width of the W-boson and  $(10.7 \pm 0.5)\%$  for the leptonic branching ratio. Adding the errors quadratically yields  $\Gamma_1^{W,\text{exp}} = (223 \pm 13)$  MeV showing that the experimental error in  $\Gamma_1^{W,\text{exp}}$  at present is more than one order of magnitude larger than the error in the leptonic Z-boson width (see (12)) and obviously much larger than the one in  $G_\mu$  (see (13)).

Even though a precise experimental input value for  $\Gamma_1^W$  is not available, it is nevertheless instructive to simulate the analysis in the  $\Gamma_1^W$ -scheme by using the theoretical SM value for  $\Gamma_1^W$  as hypothetical input parameter for evaluating (26). For the choice of  $m_t = 180$  GeV and  $M_H = 300$  GeV one obtains  $\Gamma_1^W = 226.3$  MeV as theoretical value of  $\Gamma_1^W$  in the SM. One should note that our procedure here is technically analogous to commonly used parametrizations of radiative corrections where, for instance in the on-shell scheme (see e.g. Ref. [15]), the corrections are expressed in terms of the W-boson mass  $M_{W^\pm}$ , while in an actual evaluation  $M_{W^\pm}$  is substituted by its theoretical SM value in terms of  $\alpha(M_Z^2)$ ,  $M_Z$ , and  $G_\mu$ .

In order to illustrate the fact that the replacement of the input quantity  $G_\mu$  by  $\Gamma_1^W$  indeed strongly affects the relative size of the fermionic and bosonic contributions entering each observable, we have given in Tab. 2 the relative values of the SM one-loop fermionic and bosonic corrections to the observables  $\bar{s}_W^2$ ,  $M_{W^\pm}$ , and  $\Gamma_1$  in the  $G_\mu$ -scheme and in the simulated  $\Gamma_1^W$ -scheme based on the input value  $\Gamma_1^W = 226.3$  MeV. The size of the radiative corrections in the two schemes is compared with the relative experimental error of the observables (see (12)). Table 2 shows that in the  $G_\mu$ -scheme the bosonic corrections to  $\bar{s}_W^2$  and  $\Gamma_1$  are quite sizable and considerably larger than the experimental error. In the (simulated)  $\Gamma_1^W$ -scheme, on the other hand, these corrections are smaller by an order of magnitude and have about the same size as the experimental error. The bosonic contributions to  $M_{W^\pm}$  are smaller than the experimental error in both schemes. It can furthermore be seen in Tab. 2 that the cancellation between fermionic and bosonic corrections related to the scale change is not present in the  $\Gamma_1^W$ -scheme.

The explicit values for the pure fermion-loop predictions of the observables at one loop in the simulated  $\Gamma_1^W$ -scheme read

$$\bar{s}_{W,\text{ferm}}^2 = 0.23154, \quad \left( \frac{M_{W^\pm}}{M_Z} \right)_{\text{ferm}} = 0.8813, \quad \Gamma_{1,\text{ferm}} = 84.06 \text{ MeV}. \quad (27)$$

Comparison with the experimental values of the observables given in (12) shows that there are indeed no significant deviations between the pure fermion-loop predictions in the simulated  $\Gamma_1^W$ -scheme and the data, i.e. they agree within one standard deviation. This has to be contrasted to the situation in the  $G_\mu$ -scheme, where the pure fermion-loop predictions differ from the data by several standard deviations (see (22) and Fig. 3a).

In summary, we have demonstrated that after replacing the low-energy quantity  $G_\mu$  by the high-energy observable  $\Gamma_1^W$  in the theoretical predictions for the observables  $\bar{s}_W^2$ ,  $M_{W^\pm}/M_Z$ , and  $\Gamma_1$ , no corrections beyond fermion loops are required in order to consistently describe the data. Although at present, due to the large experimental error in  $\Gamma_1^W$ , the so-defined  $\Gamma_1^W$ -scheme is of no practical use for analyzing the precision data, from a theoretical point of view it shows that the only bosonic corrections of significant magnitude are such that they can completely be absorbed by the introduction of the quantity  $\Gamma_1^W$ .

## 5 Conclusions

In this paper the nature of those electroweak bosonic loop corrections that are significant in the comparison between theory and present precision data has been investigated. The analysis has been based on the leptonic LEP1 observables  $\Gamma_1$  and  $\bar{s}_W^2$ , which are not influenced by the discrepancies noted in certain hadronic decay modes of the Z boson, and the W-boson mass  $M_{W^\pm}$ . The experimental uncertainty in the hadronic sector enters only via the input parameter  $\alpha(M_Z^2)$ . As further input parameters  $M_Z$  and  $G_\mu$  have been used.

For analyzing the structure of the bosonic corrections it is particularly convenient to use the set of effective parameters  $\Delta x$ ,  $\Delta y$  and  $\varepsilon$  being introduced on the basis of an effective Lagrangian that quantifies different sources of SU(2) violation. It has been shown that non-fermionic corrections are only required in the single parameter  $\Delta y$  which in turn is practically independent of the Higgs sector of the theory.

By studying the bosonic contributions entering  $\Delta y$  it has furthermore been pointed out that the bosonic corrections needed for an agreement between the SM predictions and the current precision data can be identified as an effect of the change in energy scale from the low-energy process muon decay to the energy scale of the LEP observables. More precisely, the bosonic corrections resolved by the precision experiments are just those furnishing the transition from the low-energy parameter  $G_\mu$  to the leptonic width of the W boson,  $\Gamma_1^W$ . Upon introducing the high-energy quantity  $\Gamma_1^W$  ( $\Gamma_1^W$ -scheme) as input parameter instead of  $G_\mu$  ( $G_\mu$ -scheme), the relevant bosonic corrections for the high-energy observables  $\bar{s}_W^2$ ,  $\Gamma_1$ , and  $M_{W^\pm}/M_Z$  can be absorbed. This illustrates that the question whether bosonic loop corrections are in fact needed in order to match theory and experiment for a certain set of observables depends on the set of input quantities, or in other words on the renormalization scheme, one is using when evaluating the theoretical predictions for the observables. Since the usefulness of  $\Gamma_1^W$  as experimental input parameter is limited at present due to the large

experimental error of the W-boson width, we have demonstrated this fact by invoking the SM theoretical value of  $\Gamma_1^W$  as input. Indeed, no further corrections beyond fermion loops are needed in this case in order to achieve agreement with the data within one standard deviation.

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